MTH 408/522: Numerical analysis

Homework II: Newton's method and its extensions

(Due 09/09/19)

Problems for turning in

1. The iteration equation for the Secant method can be written as

$$p_n = \frac{f(p_{n-1})p_{n-2} - f(p_{n-2})p_{n-1}}{f(p_{n-1}) - f(p_{n-2})}.$$

   Explain why this iteration equation is likely to be less accurate.

2. Determine the order of convergence of the following sequences.
   (a) $p_n = 1/n^k$, $k > 0$.
   (b) $p_n = 10^{-2^n}$.
   (c) $p_n = 10^{-n^k}$, $k > 0$.

3. For any $\alpha > 1$, construct a sequence $p_n \to 0$ of order α.

4. Determine, within 10^{-6}, the only negative zero and the four smallest positive zeros of the function

$$f(x) = \ln(x^2 + 1) - e^{0.4x}\cos(\pi x),$$

   which has infinitely many zeros.

Problems for practice

1. In each of the following, use the Newton-Raphson, Secant and Regula Falsi methods for finding solutions to the equation $f(x) = 0$ in the interval $[a, b]$ accurate to within ACC.
   (a) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$; $[a, b] = [0, 1]$; $ACC = 10^{-6}$.
   (b) $f(x) = x^2 - 4x + 4 - \ln(x)$; $[a, b] = [2, 4]$; $ACC = 10^{-7}$.
   (c) $f(x) = \sin(x) - e^{-x}$; $[a, b] = [3, 4]$; $ACC = 10^{-5}$.
   (d) $f(x) = 2x\cos(2x) - (x - 2)^2$; $[a, b] = [3, 4]$; $ACC = 10^{-5}$.
   (e) $f(x) = \ln(x - 1) + \cos(x - 1)$; $[a, b] = [1.3, 2]$; $ACC = 10^{-5}$.
   (f) $f(x) = x + 1 - 2\sin(\pi x)$; $[a, b] = [0.5, 1]$; $ACC = 10^{-7}$.

2. Determine the number of iterations required to find a root of $f(x) = \cos(x) - x$ within 10^{-7} with an initial approximation $p_0 = \pi/4$.